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BENG 5020

# Introduction and Background

This document outlines a model of the heat transfer that occurs inside the human lung. The mathematical relationships detail boundary conditions, initial conditions, constraints, assumptions within the scope of this model.

Heat transfer occurs in the lumen of the human lung, specifically along the biological layers mucosa and Airway Surface Liquid (ASL). The lungs are divided into an upper and lower section known as the bronchial region and the alveoli, respectively. The upper airway, specifically the trachea, bronchi, and bronchioles, conduct air transfer, while the lower airway is responsible for respiratory gas transfers1.

Within the lungs are multiple generations, nodes of dichotomous subdivisions that create a branching structure within the lungs. The 17 generations present in the bronchial region conditions the air both warm the air to body temperature and saturate the air with water1. However, the energy expenditure of warming the air can have adverse effects on the airway. Exercise-induced asthma, normal exercising, and cold air intake is known to cause ventilatory compromise2.

The process of water saturation and warming of air within the bronchial region is reversed upon expiration. Saturated air cools in the lumen and thus conserves some of the potential losses of heat and water from the ASL due to inhalation3. Thus, mass transfer of water and heat transfer within the air are relative to each other, however, the dependent system is outside the scope of this simplified model.

The purpose of this model is to compare the heat of respiration as transient condition when a significantly colder (-18.5 ˚C) ambient air temperature is introduced to the lungs. The finite model represents heat transfer in a radial fashion, extending outward of the center of the lumen (Figure 1) and axially, extending into the lumen (Figure 2). The model doesn’t consider the decreasing ASL layer as it dehydrates from water mass transfer, but rather takes into account the heat transfer due to the convective cooling as one inspires air. The hypothesis of this model is that the transient response with a colder inlet air temperature will quickly heat up to temperatures comparable with the steady state response at 22 °C. This is aided by principles of the necessity of distal bronchial generations to condition cold air4.

## SMART Goals

The execution of this project was defined with goals using a common method that uses small steps to accomplish a bigger project. The goals are specific, measurable, assignable, realistic, and timebound (SMART). This idea was pioneered by George T. Dolan in 1981 and used to help accomplish this task. The specific aspect of the goal was to create a model that would at least output a plot that could show a transient response in the heat transfer within the system of the human airway. This goal was measurable, and used a clear indicator of data values in numpy arrays to measure if a transient response was recorded by the model. The task was assigned to me, but subset assignments were made for when and where this could be accomplished, and who could assist with the troubleshooting involved. Finally, this goal is both realistic and timebound, having a deadline set by the professor, and realistic prospects, having literature articles that have reported successful implications of similar models5.

Diagram of a diagram of the lungs

Description automatically generated**Figure 1.** A tree structure of the bronchial region as well as the cross sectional view of the lumen, ASL, and mucosa layers1.

## Background, Methods, and Equations

Heat and water transfer is considered as a process of steady state airflow in the lumen of a single airway when the ambient temperature is constant. The air is assumed to be incompressible and Newtonian as assumed in previous works1. Water transfer is found to occur almost completely within the ASL layer3, (composed of mucus and the PeriCiliary Layer or PCL, which aids in mucus dispersion) and therefore only two layers are needed for heat transfer analysis: one layer accounts for the transient response of the ASL while the other layer accounts for the steady heat transfer rate without water transfer.

Heat conduction via conduction in two dimensions can be measured as:

Where *r* (m)is the radius from the center axel of the lumen, *T, t,* and *α* are the temperature, time, and thermal diffusivity, respectively. However, this equation fails to take into account the convective boundary layer at the interface at the lumen (see Figure 2), which is added back into the model by Equation 3.

For the purely diffusive portion of the heat transfer (radial), the thickness of the radial distance () that is affected by heat transfer is where and the thermal diffusivity is such that is approximately 500 µm1. This radial thickness becomes a constant boundary condition of 37 ˚C. The subsequent boundary condition for the temperature at the interface of the ASL layer is dependent on axial position, which is expressed as the energy flux by convection plus the latent heat of phase change at the lumen side (shown in Equation 7**).** Lastly, initial conditions will model a system in which a steady state is assumed between the specified body temperature (Tb = 37 ˚C) and the ambient temperature (T­a = 27.6 ˚C) throughout the whole lumen, except in the test condition at which the ambient temperature is introduced (Tb’ = -18 ˚C).

(2)

Here, Equation 2 shows the heat flux via conduction at the ASL side of the interface (r = R shown in Figure 2). is the thermal conductivity of the ASL (assumed to be equivalent to water at 0.62 W m-1 K-1), R+ and R- denoting the ASL and lumen interfaces, respectively. For the heat flux at the ASL interface (r = R) where convection governs the heat transfer coefficient (h),

(3)

A diagram of a diagram of a flow

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**Figure 2.** A representation of the lumen airflow with surrounding tissue being cooled from energy transfer into the flowing air and phase transition of water into vapor.

Therefore, to calculate a transient heat change with respect to time and space in three dimensions after exposure to cold air, certain computing advantages are used. First, the Navier-Stokes equation describes the mass transfer of air into the bronchial region and throughout the lumen (Equation 3).

Where *v* and *vT* is the kinematic viscosity of air and Vreman’s subgrid-scale eddy viscosity, respectively. This equation is simplified by Haut as

Equation 4 is solved (with *Tw* as the temperature of the lumen wall)using Equations 5 – 6 as the boundary conditions, yielding Equation 7, which then can then describe the heat flux along the axial direction of the lumen.

(5)

Where is the temperature at the wall in centigrade with respect to the axial direction, is the velocity of the inspired air, is the temperature at the airway wall, is the ambient air temperature, is the latent heat of fusion, represents the diffusivity coefficient of inspired air and water, while is the concentration of the water at the ASL interface and is the concentration of water in the inspired air. Equation 1 describes the transient response of heat flow from the surrounding tissue to the cold air in the lumen in two dimensions for every finite step along the axial direction of the lumen (dx). However, the boundary conditions of each finite step in the x direction (Equation 2) are dependent on x, and thus varies the symmetric radial conductive transfer at a single time step with respect to the x position along the lumen (Equation 7). As the time step changes, so does the mass transfer of air into the lumen, thus varying the boundary condition along the x axis. Therefore, heat transfer occurring radially is a function of the heat transfer occurring axially, including the latent heat of fusion that induces more cooling on the tissue wall and reduces the mass of the ASL layer.

By solving Equations 1 and 7, the interfacial temperature is obtained and becomes an essential boundary condition for any time step when iterating the conduction equation with respect to space in two directions. The values for certain variables are obtained from Wu et al3 shown in Table 1.

**Table 1.** Wu’s values for heat transfer values for air3.

A screenshot of a computer

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The surrounding tissue of the lumen is assumed to have constant thermal properties. Radial heat transfer as modeled in Equation 1 is rewritten as

Which then can be rewritten as a forward-time centered-space finite difference:

This model simplified the principles of the defined equations above and ultimately relied on a mass and energy transfer, as described by McFadden2 (Figure 3). Equation 10 – 15 shows the derivation of the convective condition of the air at the wall to determine the change in temperature of the innermost layer of the wall tissue.

A diagram of a bed

Description automatically generated

**Figure 3.** A defined element of space in the airway, presented by McFadden et al3.

(10)

(11)

(12)

(13)

(15)

Where is the mass flow rate (kg/s), *T(z)* is the temperature of the tissue at the axial position z, *A* is the area (m2), *h* is the heat transfer coefficient calculated by solving the Nusselt number (*N­u­)* and Prandtl number *(Pr)* that use *D* as the hydraulic diameter in meters, as the density, velocity, and viscosity of air, respectively in kg/m3, m/s, and kg/ms.

This data is modeled and compared to the results produced by Wu et al, and supplementary data produced by Haut, and McFadden1-3.

# Python Code

The code developed to model the described transient system of heat transfer in radial and axial dimensions in the airway of the lungs was modeled in python (Figure 4).

import numpy as np

import matplotlib.pyplot as plt

from matplotlib import cm

file1 = open('Final Project Tissue.csv', 'wb')

file2 = open('Final Project Data.csv', 'wb')

file1.write('Tissue\r\n'.encode())

file2.write('Axial\r\n'.encode())

def brach (T\_a):

    # define variables

    time = 2 #seconds

    r\_inner = .02 #meters

    asl\_thick = .0004 # see page 2121 in Wu for a 2s breathing period

    L\_total = .25 # meters of the whole airway and all generation of the lung

    L = L\_total /100

    k\_air = 2.68\*10\*\*-2 #J/m/s/K

    k = 0.62 # W/m/k

    c\_p\_air = 1080 #J/kg/k

    c\_p = 4180 # J/kg/k Frontiers

    rho = 993 #kg/m^3

    rho\_air = 1.2614 #kg/m^3

    visc\_air = 1.7504 \* 10 \*\*-5 #kg/ms

    alpha = k/(rho\*c\_p)

    alpha\_air = k\_air/(rho\_air\*c\_p\_air)

    thickness = np.sqrt(alpha\*time) + asl\_thick

    r\_outer = r\_inner + thickness

    print("Thickness:", thickness)

    # alpha = 3.8 \* 10 \*\* -4

    print("alpha for tissue in the lumen and the air are", alpha, ", ", alpha\_air)

    u = 5.6 # m/s (velocity)

    dr = thickness/10

    dt = dr\*\*2/(2\*alpha) # time steps at 0.1 s

    dr\_airway = 0.02/10

    dz = L/20

    # dz = dr

    T\_b = 37+273.15 # T of the body tissue further out than 500

    T\_initial = 27+273.15 # Steady state room temperature

    radial = alpha\_air \*(dt/dr\*\*2)

    axial = alpha\_air \* (dt/dz\*\*2)

    title = []

    Fo = alpha \* dt/(dz \* dz)

    print("The outer radius and the radial step respectively are,",r\_outer,dr)

    print("The axial segment size is and its steps respectively are,",L,dz)

    np.set\_printoptions(precision=15, threshold=30, edgeitems=11, suppress=True) # set print options

    # empty matrix arrays

    Tissue = np.zeros([2,int(thickness/dr+1), int(L\_total/dz+1)])

    Air\_in = np.full([1,int(r\_inner/dr\_airway + 1),int(L\_total/dz+1)], 273.15 + 22, float)

    Air = np.full([2,int(r\_inner/dr\_airway + 1),int(L\_total/dz+1)], 273.15 +22, float)

    ax, fig = plt.subplots()

    # write file def

    def writefile(i):

        file1.write(str('Tissue\r\nAt time {:.7f}\r\n'.format(i\*dt)).encode())

        np.savetxt(file1, np.atleast\_2d(Tissue[i,:,:]),fmt='%.4f', delimiter=', ', newline = '\r\n')

        file2.write(str("Tissue Air interface Temperature at time {:.7f}\r\n".format(i\*dt)).encode())

        np.savetxt(file2, np.atleast\_2d(Air[i,-1,:]),fmt='%.4f', delimiter=', ', newline = '\r\n')

        file1.write('\r'.encode())

        file2.write('\r'.encode())

    # calculate h

    def hxfer\_coeff (which,local):

        # which determines whether to use Re\_x or Re\_L

        Re\_rwall = r\_inner\*2\*u\*rho\_air/visc\_air # Reynolds number radially or x direction (used this)

        Re\_r = (local \* dr) \* u \*rho\_air/visc\_air # Reynolds local number radially

        Re\_L = u \* (local\*dz) \* rho\_air/visc\_air #Reynolds local number axially

        Pr = visc\_air \* c\_p\_air/k\_air # avg prandtl number

        Nu= 1.86 \* Re\_r\*\*(1/3) \* Pr\*\*(1/3)\*(r\_inner/2/L)

        Dittus\_Boelter = 0.023\*Re\_rwall\*\*0.8\*Pr\*\*0.3

        if which == 0:

            h = Dittus\_Boelter \*k\_air/(r\_inner\*2)

        elif which == 1:

            if Re\_r < 3\*10\*\*6:

                Nu\_r\_laminar = 0.332\*Re\_r\*\*0.5\*Pr\*\*(1/3)

                h = Nu\_r\_laminar \*k\_air/(r\_inner - local\*dr \* 2) # h is divided by the local diameter

            else:

                Nu\_r\_turb = 0.0288\*Re\_r\*\*(4/5)\*Pr\*\*(1/3)

                h = Nu\_r\_turb \* k\_air/(r\_inner - local\* dr\*2)

        else:

            if Re\_L< 3\*10\*\*6:

                Nu\_L\_laminar = 0.664\*Re\_L\*\*0.5 \* Pr\*\*(1/3)

                h = Nu\_L\_laminar \*k\_air/(local \* dz)

            else:

                Nu\_L\_turb = 0.0360\* Re\_L\*\*(4/5)\*Pr\*\*(1/3)

                h = Nu\_L\_turb \*k\_air/(local \* dz)

        return h

    # initialize the temperatures assuming equilibrium at room temperature (steady state case)

    for j in range(11):

        radius = r\_inner + j \* dr

        Tissue[:,j,:] = ((T\_b-T\_initial)/np.log(r\_outer/r\_inner))\*np.log(radius/r\_inner) + T\_initial # initialize values of radial profile in tissue

    Air\_in[0,:,0] = T\_a

    Air[1,:,:] = np.copy(Air\_in)

    writefile(0)

    Tissue\_in = Tissue[:,:,:].copy()

    print("The stability constraint for the time step is", dt)

    print("The length of T and air are",len(Air[0,:,0]), len(Air[0,0,:]))

    j, k = 1,1

    m\_dot = 2\*np.pi\*r\_inner\*u\*rho\_air

    hxfer = hxfer\_coeff(0,k)

    # main for loop for center finite difference(when introduced to cold air)

    for i in range(1,int(time/dt + 1)): # total time steps

        if i < 3:

            airhead = dt\*i\*u/dz

            if i ==2:

                airhead = 2001

            for j in range(len(Tissue[0,:,0])):

                Air[i,j,:int(airhead)] = T\_a

                airhead += -1

        for j in range(1, len(Tissue[0,:,0])-1):

            radius = j\*dr

            for k in range(1,int(L\_total/dz)):

                if j == 1:

                    q\_conv = dt\*2\*np.pi\*r\_inner \* hxfer \*dz\*(Tissue[i,0,k] - Air[i,-1,k])

                    Air\_in[0,-1,k] = q\_conv/(m\_dot \* c\_p\_air\*dt) + Air[i,-1,k-1]

                    Air\_in[0,j-1,k] = T\_a

                    Tissue\_in[1,0,k] = Tissue[i,0,k] - q\_conv\*dr/(k \*r\_inner\*2\*np.pi\*dz)

                # Transient radial and axial step for every iteration

                Tissue\_in[1,j,k] = Tissue[I,j,k] + (2\*dt\*alpha)\*((Tissue[I,j-1,k] – 2\*Tissue[I,j,k] + Tissue[I,j+1,k])/(dr\*\*2) + (Tissue[I,j,k-1] – 2\*Tissue[I,j,k] + Tissue[I,j,k+1])/(dz\*\*2) + (Tissue[I,j+1,k] – Tissue[I,j-1,k])/(2\*dr))

        Tissue = np.vstack((Tissue, np.atleast\_3d(Tissue\_in)))

        Air = np.vstack((Air, Air\_in))

        Tissue\_in[0,:,:] = Tissue\_in[1,:,:]

        # assign the end of the axial air component as the boundary condition for the subsequent step

        if I % int(time/(dt\*4)) == 0:

        # if I % 1 == 0:

            writefile(i)

            title.append(I \* dt)

    plot = True

    plot\_lin = True

    if plot\_lin == True:

        for I in range(4):

            x\_axe = [dz\*l\*100 for l in range(int(len(Tissue[0,0,:])))]

            y\_axe = [Air[I,-1,l] for l in range(int(len(Tissue[0,0,:])))]

            fig = plt.figure(1)

            ax = plt.plot(x\_axe,y\_axe)

            fig.legend()

        plt.show()

    # create a plot

    if plot == True:

        for k in range(4):

            fig = plt.figure()

            xaxis = ([])# initialize distance along the x axis (columns, i) to plot it

            yaxis = ([])# initialize distance along the y axis (rows, j) to plot it

            for I in range(int(len(Tissue[0,0,:]))): #initialize x-axis (j,columns) for plot

                xaxis.append(round(i\*dz\*100,4))

            for j in range(int(len(Tissue[0,:,0]))): #initialize y-axis (I,rows) for plot

                yaxis.append(round(j\*dr\*1000,4))

            ax = plt.axes(projection = ‘3d’)

            ax.set\_box\_aspect(aspect = (1,1,1)) #scale axes to match range

            xaxis,yaxis=np.meshgrid(xaxis,yaxis)

            z=Tissue[k,:,:]

            surf = ax.plot\_surface(xaxis,yaxis,z, rstride=1, cstride=1, cmap=cm.jet,linewidth=0, antialiased=False)

            fig.colorbar(surf)

            ax.set\_xlabel(‘z (cm)’)

            ax.set\_ylabel(‘r (mm)’)

            ax.set\_zlabel(‘Temperature (K)’)

            plt.tight\_layout()

            fig.suptitle(str(‘Radial tissue Profile ($\\mathregular{ ^{o}C}$)’ + ‘ at t = {:.2f} s’.format(title[k])))        # Plot title(“At time “, k\*dt)

            plt.tight\_layout()

        plt.show()

if \_\_name\_\_ == “\_\_main\_\_”:

    T\_a = -18 + 273.15

    brach(T\_a)

**Figure 4.** Python pseudocode used to model the current simplified model.

# Results and Data

The simplified model I created did not take into account the transient heat layer described by Wu. Furthermore, it modeled transient heat flow through the tissue surrounding the lumen, rather than the heat profile of the axial airflow at steady state. Consequently, the data presented by Wu, Haut, and McFadden differ due to the transient parameters. However, comparing the data shows errors in the current model that can be used to improve it.

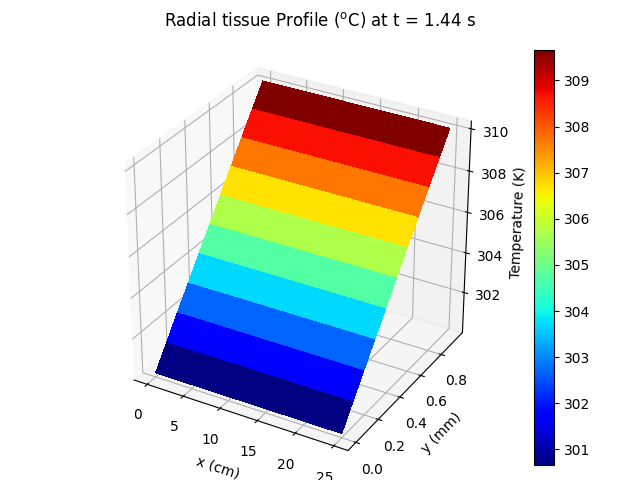
The data of the current simplified model shows the changing The temperature profile radially along the tissue at four distinct time steps (Figure 4, A – D). The plots, however, appear linear because the profiles vary by small amount along the axial direction.

A diagram of a temperature

Description automatically generated with medium confidenceA diagram of a temperature

Description automatically generated with medium confidence

1. (B)

A diagram of a temperature

Description automatically generated with medium confidence

(C) (D)

**Figure 4.** Radial temperature profiles of the cross section of the lumen (mm) throughout the axial dimension (cm) of the system. (A) shows the profile at 0.48 s. (B) shows the profile at 0.96 s. (C) shows the profile at 1.44 s. (D) shows the profile at 1.92 s.

The data provided by Wu, including the transient mucosal layer in the lumen, exhibits a temperature gradient as shown in Figure 5. This data reveals a logarithmic trajectory that approaches Tb (body temperature at 37 °C). The curve is not harshly defined, so the current simplified model (Figure 4) is consistent with Wu’s data, even though it appears linear. However,

A diagram of a curve

Description automatically generated

**Figure 5.**  Temperature profile of surrounding tissue with respect to radial distance from the lumen axis3.

More interestingly, Wu et al., McFadden et al., and Haut report the temperature of the air with respect to distance (mm) from the top of the trachea (Figure 6). These results report inspired temperatures for both one and three-dimensional CFD models, as well as McFadden’s experimental data, and are comparable to linear trend seen in Table 2.

A graph of a temperature

Description automatically generated with medium confidence

**Figure 6.** (a) Temperature (in centigrade) of the lumen air with respect to distance from the top of the trachea (mm) at a tidal volume of 15 L/min. (b) Comparable temperature plots for inspired air using data from a three-dimensional CFD computation as well as McFadden’s experimental data.

**Table 2.** Simplified current model at 1.28 mm from the top of the trachea

|  |  |
| --- | --- |
| Time (s) | Temperature (°C) |
| 0 | 22 |
| 0.48 | -18.0003 |
| 0.96 | -18.0007 |
| 1.44 | -18.0011 |
| 1.92 | -18.0015 |

# Discussion

The model for heat transfer in the tissue of the lumen due to convective heat transfer in the airway was modeled in a two-dimensional profile. Similarly, the data that Wu reports shows that the temperature of the air at the end of the airway warms up. Unfortunately, this trend is the only trend that correlates to the data provided by the authors Wu et al., Haut et al., and McFadden et al.

The data shown in Table 2 does show an increase in temperature at every axial step in space with respect to time. However, the increase, as seen in Table 2, is a finite increase of the order of 10-4. This order of magnitude of increase is much smaller than what is seen in actual data. Therefore, an error in the computational model is faulty: likely to be an error in the forward-finite step calculations on lines 122- 125 (Figure 4).

Further improvements of this model could include the mucosal layer evaporation (using the latent heat of evaporation as described by Haut et al.), the transient conduction activity that occurs in the ASL layer (as reported by Wu et al.), and comparing parameters such as the Nu (Nusselt number), Re (Reynold’s number) in local calculations at the inlet of the trachea where flow is not thermally developed, and other parameter variations. Haut et al. uses the variable β and calculates the energy transfers due to pressure-volume work as air expands in the airway. Lastly, the modeling capabilities of Python could improve the model by plotting the temperatures in in a cylindrical form to convey the temperature and axial position relationship more clearly.

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